

On the teaching and learning of physics: A Criticism and a Systemic Approach

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Abstract. The amount of published research in Physics Education Research (PER) shows, on one hand, an increasing interest in the design and development of high performance physics teaching strategies, and, on the other hand, it tries to understand plausible ways on which the brain processes scientific information so that scientific thinking skills could be taught more effectively. As physics is a subject in which mathematical and conceptual reasoning can not be separated, instructors of physics face the problem of finding suitable advice on the most effective methods of teaching physics (i.e. how much time should be spent on intuitive conceptual reasoning and how much time in developing quantitative reasoning, and how to teach both in a unitary way). In spite the important efforts made by the PER community, the published results are overwhelming and confusing for the physics instructors in the sense that the conclusions that have arisen in those articles are in some instances controversial and far from being conclusive in pointing out a particular strategy to overcome the afore mentioned problem. Accordingly, based on the analysis of published PER work, we'll be arguing that one of the major difficulties to overcome in the teaching of physics could be associated to the lack of a consistent and coherent methodological framework for teaching which integrates both aspects, conceptual and mathematical reasoning, in a systemic way of thinking. We will be presenting a set of plausible steps that could be applied to tackle the aforementioned difficulty.

—More—(15**Keywords:** Physics Education Research; Students Performance.

1 Introduction

It is not difficult to find instructors of general physics courses anxiously perusing articles published by the Physics Education Research (PER) community, searching essentially for advice on how to approach the teaching of physics effectively. Despite the demand for actually useful pedagogy of physics, PER has not produced so far any ultimately theory in this regards, and the great amount of published research on the subject, in addition of being controversial[1, 2, 3, 4, 5, 6, 7, 8, 9] might be overwhelming and even confusing to the physics instructors in the sense that being physics intrinsically a quantitative based subject, much of the recent research favors an overemphasis on qualitative (conceptual) physical aspects [10, 5, 11, 12], while standard mathematical abilities, which are crucial for understanding physical processes, are not stressed, or even taught, because, rephrasing a passage from a recent editorial[13], they interfere with the students' emerging sense of physical insight. Consequently, physics instructors face the problem of finding suitable advice on how to approach the teaching of physics in the most efficient way and an answer to the question of how much

time should be spent on intuitive conceptual reasoning and how much time in developing quantitative reasoning. Let's mention, in passing, that the aforementioned editorial[13] has risen an interesting debate[14, 15, 16] regarding the benefits and shortcomings of science education reform in the United States in relation to its influence on the development of reasoning skills on the students, expressed in the way students use or apply the learned materials in their courses. The questions raised in this debate are of interest to be taken into consideration by the PER community and/or curriculum developers, particularly by those involved on science education reform in developing countries.

On the other hand, physics instructors need also be mindful of the importance of selecting the most appropriately functional textbook, basically because innovative active-learning teaching schemes requires students to acquire basic and fundamental knowledge through reading textbooks. Correspondingly, innovative teaching strategies should be designed to help students in processing their ever thicker and heavier textbooks, which are laden with physical and mathematical insights[17, 5, 18, 19, 20]. Thus, the panorama regarding the learning of physics is even more dramatic on the side of the students. For one reason, in their struggle to fully participate in the process of learning, at the moment of trying to find suitable learning materials that could help them to go beyond classroom instruction (i.e. aiming to develop self-confidence on their own through exercising their role of active-learners), students face the dilemma of deciding which textbook could be helpful: perhaps a conceptual physics textbook (i.e. [21]), the student could wonder; or may be a calculus based physics textbook (i.e. [22]); or why not an algebra based physics textbook (i.e. [23]); or what about a combination of all of them? could finally the student ponder scrabbling in his/her pocket/handbag to verify if the money in there could be enough to bring some extra weight at home (for a good account of the drama of choosing a textbook see for instance [24] and references there in). For another, in a typical course work for students majoring in science and/or engineering they usually must take more than one physics class. It could happen that in one term his/her physics instructor may emphasize quantitative reasoning over conceptual analysis, and in another term the respective instructor could rather accentuate conceptual learning over quantitative analysis, likely causing confusion for students, leading them to wonder which emphasis is correct.

Finally, it is not difficult to find published results by the PER community on which it is shown directly or indirectly the inability of students to express, interpret, and manipulate physical results in mathematical terms. That is, students shows a clear deficiency in their training to exploit the mathematical solution of a problem (which sometimes could be obtained mechanically or by rote procedures) to enhance their knowledge regarding conceptual physics [2, 3, 25, 11, 26, 27, 28]. More important, the analysis of published excerpts of student's responses to interviews conducted by some researchers to further understand students' way of reasoning while solving physics problems, shows that students lack of a structured methodology for solving physics problems [2, 26, 11, 12]. These findings can not be surprising at all. In fact, none of the most commonly recommended physics textbooks (i.e. [22, 29, 30]) make use of a consistently and clear problem-solving methodology when presenting the solution of the textbook worked out illustrative examples [31]. Moreover, the lack of a coherent problem-solving strategy can also be found in both the student and instructor manual solutions that usually accompany textbooks. Generally, standard textbooks problem-solving strategies encourage the use of a formula based scheme as compiled by the

formulae summary found at the end of each chapter of the text, and this strategy seems to be spread out even in classroom teaching [32]. Consequently, students merely imitates the way in which problems are handled in the textbooks, which is also perhaps the same way in which problems are solved by the instructor in class. For support of the previous assertion, one needs only browse the Internet for introductory physics courses and skim the solutions of problems posted by the course instructor. Furthermore, from the aforementioned students interviews excerpts one can also appreciate the lack of reasoning skills trying to associate or connect a way to solving a problem with the solution of other previously similar problem from another context (i.e. by using analogies). Again, the absence of this skill can not be surprising at all because students are just mirroring the unrelatedness way on which commonly used physics textbooks present the themes (i. e. the use of analogies are not fostered) [33, 34, 35].

2 On the importance of a structured, systemic methodology to solve physics problems

To further motivate the subsequent discussion, let us summarize our introductory commentaries. We are essentially pointing out three major problems in the learning and teaching of physics: 1) the demand of the physics instructors for effective teaching strategies that explain how much time should be spent on teaching intuitive conceptual reasoning and how much time on developing students' quantitative reasoning, and how to teach both aspects holistically 2) the students' need for suitable textbooks that will help them develop mathematical abilities reasoning, which are essential for enhancing their knowledge of conceptual physics, and 3) a deficiency in the teaching of physics leading to students not being taught a coherent physics problem-solving strategy that enables them to engage in both mathematical and conceptual reasoning.

A moment of thought about the above summarized difficulties leads us to postulate the necessity of a systemic [36, 37] approach which, from an operational point of view, could help instructors and students to achieve a better performance in the process of teaching and learning physics.

In a broad sense, a systemic approach in the learning and teaching of physics could be represented as a framework involving the ordered triple composition-environment-structure together with a mechanism or *modus operandi* which integrates the teaching and learning process according to an approach allowing us to tackle the aforementioned difficulties from an efficient and unifying point of view (for more details refers to Bunge [36, 37]).

On the instructor side the need of a systemics approach in the teaching of physics could be justified by the advantage of using a methodology which would help them to incorporate both conceptual and mathematical reasoning systematically in their teaching. In this way, students will obtain the necessary training in their computational skills while learning how to use mathematical formulae to obtain the physics in the equations, even when they can obtain the mathematical solutions of a problem by rote procedures. In other words, students could apply "higher order thinking skills." [38] via the mathematical understanding of a physics problem, which in turns involves meaningful learning which goes beyond the merely application of rote procedures. Moreover, using properly designed quantitative problems that require students to

illustrate their conceptual learning and understanding will reveal much to instructors about their students' learning and will provide invaluable feedback[39, 40, 17, 41], and such problems can also be a powerful way to help students to understand the concepts of physics[38, 41], a point emphasized by the Nobel prize-winning, great physicist Lev Davidovich Landau on the importance of first mastering the techniques of working in the field of interest because "fine points will come by itself." In Landau's words, "You must start with mathematics which, you know, is the foundation of our science. [...] Bear in mind that by 'knowledge of mathematics' we mean not just all kinds of theorems, but a practical ability to integrate and to solve in quadratures ordinary differential equations, etc." [42] To further enhance their reasoning skills, the students would have the opportunity to increase their intuitive conceptual skills in the physics laboratory, where conceptual learning is reinforced by experience[43, 44].

On the student side, the need of a systemics approach in the learning of physics could be justified by the usefulness of applying a working methodology which could help them to approach the learning of physics from a interrelate point of view. That is, that his/her knowledge of mathematics is useful to master ideas from physics, and that the use of analogies are important to approach the solution of physical, mathematical and engineering problems. In short, this kind of practical, unified problem-solving strategy will help students to pose and approach any kind of problem, and will teach students that physics is the primary subject to start developing these kind of analytical skills. In other words, with such an approach, students would internalize that it is in physics classes where they can start to apply what they have learned in their math classes and to find new non-formal approaches to performing computations[45]. One could resort to the anecdote of the cathedral building[46, 47] to further illustrate the need of keeping in mind the interconnectedness of reasoning skill, mathematics and physics. To paraphrase Heron and Meltzer, learning to approach problems in a systematic way starts from teaching and learning the interrelationships among conceptual knowledge, mathematical skills and logical reasoning[48]. An example of this assertion is illustrated by an explanation of what happened to the Millennium Bridge disaster, . which stated "Existing theories of what happened on the bridge's opening day focus on the wobbling of the bridge but have not addressed the crowd-synchronization dynamics. In our approach, wobbling and synchrony are inseparable. They emerge together, as dual aspects of a single instability mechanism, once the crowd reaches a critical size." [49] The mistakes made in the construction of the Millennium Bridge help us to understand the need for teaching and learning based on a systemic approach which recognizes the interrelatedness of every aspect of the physical process (physics, mathematics, and engineering design).

3 A systemic structured methodology to solve physics problems

Earlier work on the importance and necessity of a problem-solving strategy can be found in the work of the great mathematician George Pólya [50, 51], who made emphasis on the relevance of the systematicity of a problem-solving strategy for productive thinking, discovery and invention. Some of his views, either provocative or encouraging, about teaching and learning can be found spread out in some PER publications, like for instance that *teaching is not a science* (i. e. [2]); on *the aim of teaching* (i. e. [38, 6]); that *teaching is an art* (i.

e. [52]); on the importance of *problem-solving skills* (i. e. [39, 40, 3, 53]), etc. For a further detailed account of Pólya's work let's refers to [50, 51, 54, 55, 56, 57].

In *How to Solve It*, Pólya set four general steps to be followed as a problem-solving strategy:

- P1 **Understanding the problem:** some considerations to develop at this step involves drawing a figure and asking questions like What is the unknown? What is the condition? Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory? Draw a figure. Introduce suitable notation.
- P2 **Devising a plan:** some considerations to have in mind in order to develop this step involves looking at the unknown and trying to think of a familiar problem having the same or a similar unknown. Some questions to be ask are like Have you seen this before? Or have you seen the same problem in a slightly different form?
- P3 **Carrying out the plan:** Be sure to check each step and make sure that the steps are correct.
- P4 **Looking back:** some considerations to develop at this step involves asking questions like Can the results be checked? Can the results be derived differently? Can the result or the method be applied to solve or fully understand other problems?

Surprisingly, these steps encompass “the mental processes and unconscious questions experts explore as they themselves approach problem solving.”[57] These four step are also the base of some computational models devised to “model and explore scientific discovery processes.”[57]

Now, even though the aforementioned four steps seems very simply, their generality seems to be hard for novices to follow them. Thus, in order to have a problem-solving strategy more affordable to students we extended the four steps problem-solving strategy into a six steps strategy. We made our choice based on empirical observations after experimenting with a five step strategy reported in [53]. Justification for a more detailed problem-solving strategy can be found in the words of Schoenfeld “First, the strategies are more complex than their simple descriptions would seem to indicate. If we want students to use them, we must describe them in detail and teach them with the same seriousness that we would teach any other mathematics.”[58] Nevertheless, we will further rationalize below the need for explicitly including a new step in our proposed methodology (see item 5). Thus, our six steps proposed strategy is as follows:

1. **Understand the problem:** in this stage students needs to actually be sure to what the problem is. In addition to making drawings to get a grasp of the problem formulation, eventually they might need to reformulate the problem in their own words, making that that they are obtaining all the giving information for solving the problem. This is a crucial step in the sense that *if one does not know where are we going, any route will take us there*.
2. **Provide a qualitative description of the problem:** in this stage students needs to think and write down the laws, principles or possible formulations that could help them

to solve the problem. For instance students need to consider any possible frame work of analysis that could help them to represent or describe the problem in terms of the principles of physics (i.e. Newton's law, energy conservation, momentum conservation, theorem of parallel axis for computing inertia moment, non-inertial reference system, etc.) If necessary, the drawings of the previous step could be complemented by the corresponding free-body and/or vector diagram.

3. **Plan a solution:** once the student has as many possibilities to approach the problem, he/she only needs to pick one strategy of solution and write down the corresponding mathematical formulation of the problem, avoiding as much as possible to plug numbers in the respective equations. Also, they need to think whether the problem at hand is similar to a previously solved one, and find out whether the information at hand would be enough to get a solution (i.e. if a set of algebraic equations is under or over determined or the number of provided boundary conditions are enough to solve a differential equation). Eventually, one might need to go back to the previous step in order to get a well posed problem, perhaps by choosing another strategy.
4. **Carrying out the plan:** at this stage the student will try to find a solution to the mathematical formulation of the problem and perhaps they will need to go back in order to find easier mathematical formulations of the problem. This is facilitated if the students have written down alternatives of solution as they were supposed to do on item 2.
5. **Verify the internal consistency and coherence of the used equations:** at the moment of finding a solution of the involved mathematical equations, students need to verify whether the equations are consistent with what they represent and that the equations are dimensionally correct. Though this seems to be an unnecessary step, experience shows that the students too often do not verify the consistency and coherence of the equations they solve. And this mistake is also found to be performed by textbook writers, as discussed in a recent editorial [59]. After verifying no mistakes or inconsistencies are found in the mathematical solution of the problem, students could then plug numbers in the obtained results to find, whether required or not, a numerical solution which in turn could be used in the next step to further evaluate the obtained result. In the provided illustrative example will show how a right answer could be obtained, though the internal consistency of a used equation is not right [60].
6. **Check and evaluate the obtained solution:** once a solution has been obtained, its plausibility needs to be evaluated. Some questions could be asked in this regard: can the results be derived differently? Can the solution be used to write down the solution of a less general problem? Can the solution be used to further understand the qualitative behavior of the problem? Is it possible to have a division by zero by changing a given parameter? Does it make sense?, and so on.

A first comment on our six steps problem-solving strategy is that it provides a unified, systemic way of approaching the solution of a physical problem encompassing both qualitative (steps 1-3) and quantitative (steps 4-6) reasoning, and instructors could make as much

emphasis as they prefer on any of the set of steps, providing the students with a structured recipe on how to approach in detail the other side of the problem's solution. Second, comparing our six steps problem-solving strategy with Pólya's four steps ones, one could see that we have explicitly divided Pólya's step one (P1) into two steps (1-2), and Pólya's step three (P3) into two steps (4-5). A further comment on this problem-solving strategy is that we prefer to call the the second step (2) *Provide a qualitative description of the problem* rather than *Physics description* as in [53], because one share the idea that students tend to think that by providing a qualitative analysis of a problem they are also providing the solution required by a physicist, and that the mathematical solution of the problem is just uninteresting mathematics. Instead, we make emphasis in that a physical solution of a problem is a combination of both qualitative and quantitative reasoning. As stressed by the great physicist Lord Kelvin "I often say that when you can measure something and express it in numbers, you know something about it. When you can not measure it, when you can not express it in numbers, your knowledge is of a meager and unsatisfactory kind. It may be the beginning of knowledge, but you have scarcely in your thoughts advanced to the state of science, whatever it may be." [21] Freeman Dyson was more eloquent "...mathematics is not just a tool by means of which phenomena can be calculated; it is the main source of concepts and principles by means of which new theories can be created." [61]

4 Illustrative Example

In the following example we will present an approach on how to introduce students in the use of our proposed six steps problem-solving strategy. Though each one of the steps has its importance, we will provide further evidence of why step five needs to be taught explicitly. It is pertinent to point out that, paraphrasing Pólya's words, by proper training students could absorb the steps of our problem-solving strategy in such a way that they could perform the corresponding operations mentally, naturally, and vigorously. The dynamics of teaching is left to the instructor. In this article we are not pretending to show how the teaching should be carried out. Innovative teaching strategies can be found elsewhere [17, 5, 18, 19, 20, 53, 62].

4.1 The problem statement

Problem: About its central axis, find the moment of inertia of a thin hollow right circular cone with radius R , lateral length L , and mass M uniformly distributed on its surface with density σ .

1. **Understand the problem:** "It is foolish to answer a question that you do not understand. ... But he should not only understand it, he should also desire its solution." [50] Following Pólya's commentary, before attempting to solve this problem, students need to have been exposed to a basic theory on computing moment of inertia (I). Particularly, students need to be familiar with the computation of I for a thin circular ring about its main symmetric axis. To further understands the geometry of the present problem, students could, for example, be talked about the shape of an empty ice cream cone. After some discussion, a drawing better than the one shown in figure 1 could be

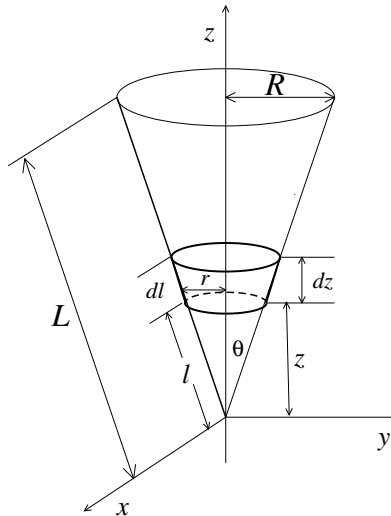


Figure 1: a hollow right circular cone with radius R , lateral length L , and uniform mass M . The cone's high is $H = \sqrt{L^2 + R^2}$. The figure also shows at the lateral distance l , measured from the cone's apex at the origin of the coordinate system, an infinitesimal ring of lateral length dl and radius r . Two useful geometrical relations among some of the dimensions shown in the figure are $r = (R/L)l$ and $r = (R/H)z$.

presented on the board. Let's mention that additional ways of presenting each step in meaningful ways can be found in [50, 53].

2. **Provide a qualitative description of the problem:** In this step one could further motivate the discussion by associating the computation of I with rotational motion quantities (i.e. kinetic energy, angular momentum, torque, etc.). One can even motivate the qualitative discussion by considering the hollow cone as a first crude approximation of a symmetric top or of a cone concrete mixer. The drawing of figure 1 could even be made more explicative.
3. **Plan a solution:** "We have a plan when we know, or know at least in outline, which calculations, computations, or constructions we have to perform in order to obtain the unknown. ... We know, of course, that it is hard to have a good idea if we have little knowledge of the subject, and impossible to have it if we have no knowledge. ... Mere remembering is not enough for a good idea, but we can not have any good idea without recollecting some pertinent facts." [50] Accordingly, at this stage instructors could point out the superposition principle to solve the problem by slicing the hollow cone in a set of small, infinitesimal, rings distributed along the symmetrical axis of the cone. Thus, each infinitesimal ring will have in common the same rotational axis about which the moment of inertia of them is already known $dI = r^2 dm = r^2 \sigma dS$, where r is the radius of each ring, while dS represents the respective infinitesimal surface of each ring.
4. **Carrying out the plan:** To carried out the plan, it won't be a surprise to choose the wrong dS . In fact, it is not difficult, at first sight, to choose wrongly (see figure 1): $dS = 2\pi r dz = 2\pi(R/H)z dz$, which lets to $S = \pi RH$, as the hollow cone surface (this

result is of course wrong). Using this surface element, the moment of inertia for the small ring takes the form $dI = 2\pi\sigma(H/R)r^3dr$, which leads to $I = 2\pi\sigma(H/R)(R^4/4) = (1/2)(\sigma S)R^2 = MR^2/2$, as the required moment of inertia of the hollow cone (which is the right answer). It is not difficult to get students performing this sort of computations and they become uneasy when trying to convince them that in spite of having found a right result, it is specious because it was obtained via a wrong choice for dS . Eventually students might agree on the incorrectness of their procedure if asked to compute explicitly the cone's mass.

5. **Verify the internal consistency and coherence of the used equations:** “Check each step. Can you see clearly that the step is correct? Can you prove that it is correct? ... Many mistakes can be avoided if, carrying out his plan, the student check each step.” [50] Steeping on our teaching experience, it is too easy for students to perform without hesitation the just aforementioned wrong computations, as presented in the previous step. And it is not easy to get students to realize their mistake. For god shake, they have computed the right answer!!!: for a hollow thin cone, rotating about its symmetric axis, $I = MR^2/2$!!! In this situation, to make aware students of their mistake, the easy way is the experiment. Instructors could unfold several hollow cones to actually show the students that the respective surface is $S = \pi RL$, instead of the wrongly obtained $S = \pi RH$. Accordingly, we hope to have provided enough evidence for the need to, explicitly and repeatedly, mention to students on the need to check each computational step, including checking for dimensionality correctness. In this case, the right approach is to consider $dS = 2\pi rdl = 2\pi l(R/L)dl$, which yield $S = \pi RL$, the right answer for S . This choice for dS leads to $dI = 2\pi\sigma(L/R)r^3dr$, which yields $I = 2\pi\sigma(L/R)(R^4/4) = (1/2)(\sigma S)R^2 = MR^2/2$, the right answer.

Considering that it is not hard to find stories on reported wrong results due to wrong or incomplete computations [49, 63], this problem could also be used as an example of how computations of a physical quantity (the surface of a cone shell) can be used to judge a mathematical result (the wrong value for S) that is used in additional computations yielding a right answer..

6. **Check and evaluate the obtained solution:** “Some of the best effects may be lost if the student fails to reexamine and to reconsider the completed solution.” [50] After gaining confidence on the obtained solution of the problem, it is necessary to spend sometime in evaluating its plausibility. Examining the solution of our problem one could ask: it is not striking that the rotational inertia for a hollow cone about its symmetric axis is the same as for a solid disk having the same uniformly distributed mass M and radius equal to the cone's base? Does not it a counter example for the statement that rotational inertia only depends on how the mass is distributed around the axis of rotation? Furthermore, if for some reason the wrong choice for the dS was not caught in the previous step, it could be detected if analyzing the case of having a non constant σ . A further interpretation of the result can be found at [64].

5 Concluding remarks

This article presents a six step problem-solving strategy, aiming to approach three major problems in the learning and teaching of physics: 1) the demand of the physics instructors for effective teaching strategies that could help in the teaching of intuitive conceptual and quantitative reasoning, and how to teach both aspects holistically 2) the students' need for suitable methodology that could help students to fill the textbooks' gap on enhancing their mathematical reasoning abilities, which are essential for reinforcing students' knowledge of conceptual physics, and 3) a deficiency in the teaching of physics leading to students not being taught a coherent physics problem-solving strategy that enables them to engage in both mathematical and conceptual reasoning.

Let's finish by recalling a particular point of view which the great mathematician Pólya stressed very much in his writings about the art of teaching and learning, which, in some sense, can be considered as an "axiomatic thought" about the art of teaching and learning. He was emphatic on the fact that "*for efficient learning, the learner should be interested in the material to be learnt and find pleasure in the activity of learning.*" In order to reinforce the content of this expression one might recall the story of the Cathedral's construction workers [47]. In other words, inspiration to learn is without doubt a necessary condition in order to have an efficient and effective teaching and learning environment. This, of course, is by no means a new discovery, and, paraphrasing Schoenfeld [65], some ideas to circumventing few of the barriers between the dedicated instructor and his/her students' attitudes in "learning" the subject that is being taught has been set forward in [65, 66, 67]. Nevertheless, one should keep in mind that "we know from painful experience that a perfectly unambiguous and correct exposition can be far from satisfactory and may appear uninspiring, tiresome or disappointing, even if the subject-matter presented is interesting in itself. The most conspicuous blemish of an otherwise acceptable presentation is the 'deus ex machina'." [68]

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References

- [1] B. Thacker, E. Kim, K. Trefz, and S. M. Lea. Comparing problem solving performance of physics students in inquire-based and traditional introductory physics courses. *Am. J. Phys.*, 62:627–633, 1994.
- [2] D. Hammer. More than misconceptions: Multiple perspectives on student knowledge and reasoning, and an appropriate role for education research. *Am. J. Phys.*, 64:1316–1325, 1996.
- [3] R. Ehrlich. How do we know if we are doing a good job in physics teaching? *Am. J. Phys.*, 70:24–29, 2002.

- [4] D. E. Meltzer. The relationship between mathematics preparation and conceptual learning gains in physics: A possible “hidden variable” in diagnostic pretest scores. *Am. J. Phys.*, 70:1259–1268, 2002.
- [5] C. Hoellwarth, M. J. Moelter, and R. D. Knight. A direct comparison of conceptual learning and problem solving ability in traditional and studio style classrooms. *Am. J. Phys.*, 73:459–462, 2005.
- [6] E. F. Redish. Changing Student Ways of Knowing: What should our students learn in a physics class? In *Proceedings of the conference, World View on Physics Education 2005: Focusing on Change, New Delhi, India*. World Scientific Publishing Co., 2005. In press. Available from <http://www.physics.umd.edu/perg/papers/redish/IndiaPlen.pdf>.
- [7] S. Ates and E. Cataloglu. The effects of students’ reasoning abilities on conceptual understandings and problem-solving skills in introductory mechanics. *Eur. J. Phys.*, 28:1161–1171, 2007.
- [8] V. P. Coletta, J. A. Phillips, A. Savinainen, and J. J. Steinert. Comment on ‘the effects of students’ reasoning abilities on conceptual understandings and problem-solving skills in introductory mechanics’. *Eur. J. Phys.*, 29:L25–L27, 2008.
- [9] S. Ates and E. Cataloglu. Reply to ‘comment on “the effects of students’ reasoning abilities on conceptual understandings and problem-solving skills in introductory mechanics”’. *Eur. J. Phys.*, 29:L29–L31, 2008.
- [10] R. Mualem and B. S. Eylon. ‘physics with a smile’-explaining phenomena with a qualitative problem-solving strategy. *Phys. Teach.*, 45:158–163, 2007. See also references there in.
- [11] M. S. Sabella and E. F. Redish. Knowledge organization and activation in physics problem solving. *Am. J. Phys.*, 75:1017–1029, 2007.
- [12] L. N. Walsh, R. G. Howard, and B. Bowe. Phenomenographic study of students’ problem solving approaches in physics. *Phys. Rev. ST Phys. Educ. Res.*, 3:020108:1–12, 2007.
- [13] D. Klein. School math books, nonsense, and the national science foundation. *Am. J. Phys.*, 75:101–102, 2007.
- [14] L. J. Atkins. Comment on “School math books, nonsense, and the National Science Foundation,” by David Klein[13]. *Am. J. Phys.*, 75:773–775, 2007.
- [15] T. Millar. Comment on “School math books, nonsense, and the National Science Foundation,” by David Klein[13]. *Am. J. Phys.*, 75:775–776, 2007.
- [16] D. Klein. Reply to comments on “School math books, nonsense, and the National Science Foundation,” by David Klein[13]. *Am. J. Phys.*, 75:776–778, 2007.
- [17] K. C. Yap and C. L. Wong. Assessing conceptual learning from quantitative problem solving of a plane mirror problem. *Phys. Educ.*, 42:50–55, 2007.

- [18] C. H. Crouch and E. Mazur. Peer instruction: ten years of experience and results. *Am. J. Phys.*, 69:970–977, 2001.
- [19] M. C. James. The effect of grading incentive on student discourse in peer instruction. *Am. J. Phys.*, 74:689–691, 2006.
- [20] W. Cerbin and B. Kopp. Lesson study as a model for building pedagogical knowledge and improving teaching. *IJTLHE*, 18:250–257, 2006. Available at <http://www.isetl.org/ijtlhe/>.
- [21] P. G. Hewitt. *Conceptual Physics*. Addison-Wesley, 7th. edition, 1993.
- [22] D. Halliday, R. Resnick, and J. Walker. *Fundamentals of Physics*. John Wiley & Sons, 6th edition, 2000.
- [23] D. Giancoli. *Physics Principles with Applications: International Edition*. Pearson Education, 6th. edition, 2004.
- [24] L. S. Dake. Student selection of the textbook for an introductory physics course. *Phys. Teach.*, 45:416–419, 2007.
- [25] M. E. Loverude, C. H. Kautz, and P. R. L. Heron. Helping students develop an understanding of archimedes’ principle. i. research on student understanding. *Am. J. Phys.*, 71:1178–1187, 2003.
- [26] L. G. Rimoldini and C. Singh. Student understanding of rotational and rolling motion concepts. *Phys. Rev. ST. Phys. Educ. Res.*, 1:010102:1–9, 2005.
- [27] D. C. Meredith and K. A. Marrongelle. How students use mathematical resources in an electrostatics context. *Am. J. Phys.*, 76:570–578, 2008.
- [28] S. Rojas. On the need to enhance physical insight via mathematical reasoning. *Rev. Mex. Fís. E*, 54:75–80, 2008.
- [29] P. A. Tipler and G. Mosca. *Physics for Scientists and Engineers*. WH Freeman and Co, 5th edition, 2003.
- [30] R. A. Serway and J. W. Jewett. *Physics for Scientists and Engineers*. Thomson Learning, 6th. edition, 2003.
- [31] Just to mention an example, see for instance the way on which [29] handle the solution of example 8-4 on page 220, as compared by the solution of any other example.
- [32] K. M. Hamed. Do you prefer to have the text or a sheet with your physics exams? *Phys. Teach.*, 46:290–293, 2008.
- [33] M. B. Schneider. Discovery-based gauss’s law. *Am. J. Phys.*, 72:1272–1275, 2004.
- [34] J. P. Donohoe. A laboratory experiment to demonstrate gauss’s law for electric fields. *Am. J. Phys.*, 76:963–967, 2008.

- [35] Just to mention an example, see for instance the way on which [29] deals with the subject of computing center of mass and rotational inertia of linear, superficial and volumetric mass distributions. Even though the techniques of solving these problems have some similarities, the textbook has not mention of it. Another example is the computation of gravitational and electric field. No where in the book is mentioned that the techniques for one case could be applied for the other and that students could reinforce their understanding and computational skills by looking at worked out examples in both sections.
- [36] M. Bunge. Systemism: the alternative to individualism and holism. *Journal of Socio-Economics*, 29:147–157, 2000.
- [37] M. Bunge. How does it work?: The search for explanatory mechanisms. *Philosophy of the Social Sciences*, 34:182 – 210, 2004.
- [38] J. S. Rigden. Problem-solving skill: What does it mean? *Am. J. Phys.*, 55:877, 1987.
- [39] F. Reif. Teaching problem solving – a scientific approach. *Phys. Teach.*, 19:310–316, 1981. See also references there in.
- [40] F. Reif and L. A. Scott. Teaching scientific thinking skills: Students and computers coaching each other. *Am. J. Phys.*, 67:819–831, 1999. See also references there in.
- [41] J. W. Dunn and J. Barbanel. One model for an integrated math/physics course focusing on electricity and magnetism and related calculus topics. *Am. J. Phys.*, 68:749–757, 2000.
- [42] E. M. Lifshitz. L. d. landau’s plain talk to students of physics. *Am. J. Phys.*, 45:415–422, 1977.
- [43] C. v. Aufschnaiter and S. v. Aufschnaiter. University students’ activities, thinking and learning during laboratory work. *Eur. J. Phys.*, 28:S51–S60, 2007.
- [44] M. Hanif, P. H. Sneddon, F. M. Al-Ahmadi, and N. Reid. The perceptions, views and opinions of university students about physics learning during undergraduate laboratory work. *Eur. J. Phys.*, 30:85–96, 2009.
- [45] F. R. Yeatts and J. R. Hundhausen. Calculus and physics: challenges at the interface. *Am. J. Phys.*, 60:716–721, 1992.
- [46] J. P. Girard and S. Lambert. The story of knowledge: Writing stories that guide organisations into the future. *The Electronic Journal of Knowledge Management*, 5:161–172, 2007.
- [47] The story goes like this. A strange fellow approached three construction workers who were part of the team building a cathedral. “Excuse me, young man, what are you doing?” the strange guy asked one of the workers, who was doing his work with some anxiety. “I am cutting stones,” the worker replied. Realizing that the construction worker had no desire to engage in conversation, the strange man repeated his question to another

worker. This time, this construction worker paused and stated that he was a stonecutter from the north who had come to work and who would return home as soon as he had earned some money. After thanking the second worker, the strange man approached the third construction worker, who was performing his work with enthusiasm, patiently and carefully making and remaking what he was doing, and again asked his question. This construction worker completely stopped what he was doing as if to begin a long discussion with the strange man, and answered: "I have journeyed many miles to be part of the team that is constructing this magnificent cathedral. Even though I have spent many months away from my family and I miss them dearly, I am aware of how important this cathedral will be one day and I know how many people will find spiritual peace in this great place." This worker continued his story, telling the strange man about the planning of the construction of the cathedral, and how the construction would be completed, and so on. Relating the anecdote with students' behavior, the first worker appears like the students who painfully are taking a course and only want a passing grade. The second worker is like the students who are doing just enough to obtain their passing grade. The third worker is like the active-learner students, who have realized that their university life is much more than obtaining a passing grade, and that every subject is interconnected each to another. Thus, several perspectives can be applied to study any subject.

- [48] P. R. L. Heron and D. E. Meltzer. The future of physics education research: intellectual challenges and practical concerns. *Am. J. Phys.*, 73:390–394, 2005.
- [49] S. H. Strogatz, D. M. Abrams, A. McRobie, B. Eckhardt, and E. Ott. Crowd synchrony on the millennium bridge. *Nature*, 438:43–44, 2005.
- [50] G. Pólya. *How to Solve it. A new aspect of mathematical method*. Princeton University Press, Inc., 2nd. ed., second printing. edition, 1973.
- [51] G. Pólya. On learning, teaching, and learning teaching. *The American Mathematical Monthly*, 70:605–619, 1963.
- [52] M. Milner-Bolotin. Teachers as actors: The implications of acting on physics teaching. *Phys. Teach.*, 45:459–461, 2007.
- [53] P. Heller, R. Keith, and S. Anderson. Teaching problem solving through cooperative grouping. part 1: Group versus individual problem solving. *Am. J. Phys.*, 60:627–636, 1992.
- [54] A. H. Schoenfeld. *Mathematical Problem Solving*. Academic Press, Inc., 1985.
- [55] A. H. Schoenfeld. Pólya, problem solving, and education. *Mathematics Magazine*, 60:283–291, 1987.
- [56] A. H. Schoenfeld. Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In *D. Grouws (Ed.), Handbook for Research on Mathematics Teaching and Learning*, pages 334–370. MacMillan, 1992.

- [57] E. Lederman. Journey into problem solving: A gift from polya. *Phys. Teach.*, 47:94–97, 2009.
- [58] A. H. Schoenfeld. Teaching problem-solving skills. *The American Mathematical Monthly*, 87:794–805, 1980.
- [59] C. F. Bohren. Physics textbook writing: Medieval, monastic mimicry. *Am. J. Phys.*, 77:101–103, 2009.
- [60] In order to further show students the necessity of continuously verifying the consistency of the used equations, one could resort to the naive problem involving the wrong proof that $1 = 2$: $x \times x - x \times x = x^2 - x^x \rightarrow x(x - x) = (x - x)(x + x) \rightarrow$ (after cancelling (x-x) in both sides) $x = 2x \rightarrow 1 = 2$.
- [61] F. J. Dyson. Mathematics in the physical sciences. *Scientific American*, 211:128–145, 1964.
- [62] P. Heller and M. Hollabaugh. Teaching problem solving through cooperative grouping. part 2: Designing problems and structuring groups. *Am. J. Phys.*, 60:637–644, 1992.
- [63] J. Veysey and II N. Goldenfeld. Simple viscous flows: From boundary layers to the renormalization group. *Rev. Mod. Phys.*, 79:883–927, 2007.
- [64] D. Bolam and I. Wilkinson. Inertia of a conical shell. *The mathematical Gazette*, 45:335–336, 1961.
- [65] A. H. Schoenfeld. Some techniques for the mathematics classroom. *The American Mathematical Monthly*, 84:52–53, 1977.
- [66] R. Ehrlich. Giving bonus points based on oral exams. *Am. J. Phys.*, 75:374–376, 2007.
- [67] G. Duda and K. Garrett. Blogging in the physics classroom: A research-based approach to shaping students’ attitudes towards physics. *Am. J. Phys.*, 76:1054–1065, 2008.
- [68] G. Pólya. With, or without, motivation? *The American Mathematical Monthly*, 56:684–691, 1963.

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